



**Dept. of Electrical Engineering**  
**First Exam, First Semester: 2017/2018**

**Course Title: Power Systems 2**

**Date: 22/11/2017**

**Course No: (610412)**

**Time Allowed: 50 Minutes**

**Lecturer: Dr. Mohammad Abu-Naser**

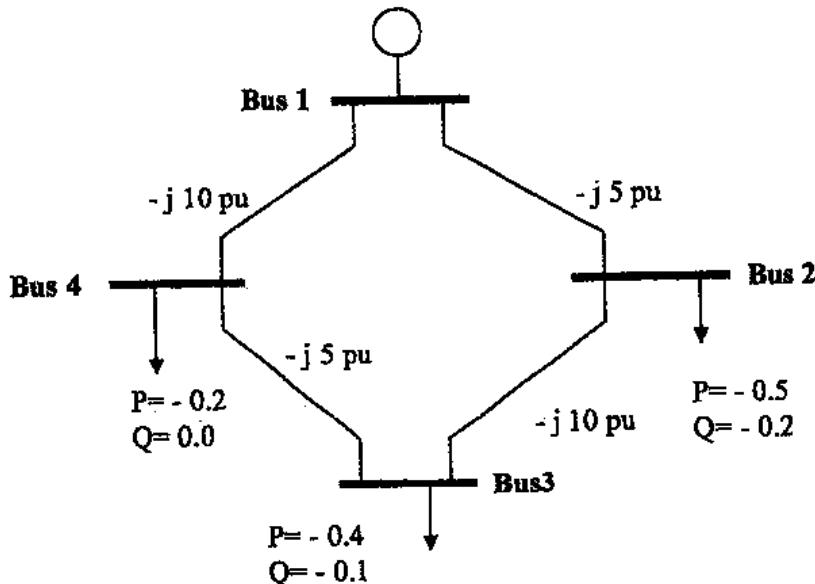
**No. of Pages: 2**

**Question 1: (10Mark)**

**Objectives: This question is related to load flow analysis using GS method**

In the following power system, branch admittances and bus loads are given in per unit.

- Construct the bus admittance matrix.
- Using Gauss-Seidel method, determine voltages at buses 2, 3, 4 for the first iteration. Bus 1 is slack with  $V_1 = 1\angle 0^\circ$ . Start with an initial estimate of  $V_2^{(0)} = 1\angle 0^\circ$ ,  $V_3^{(0)} = 1\angle 0^\circ$ ,  $V_4^{(0)} = 1\angle 0^\circ$ .



**Question 2: (10Mark)**

**Objectives: This question is related to load flow analysis using NR method**

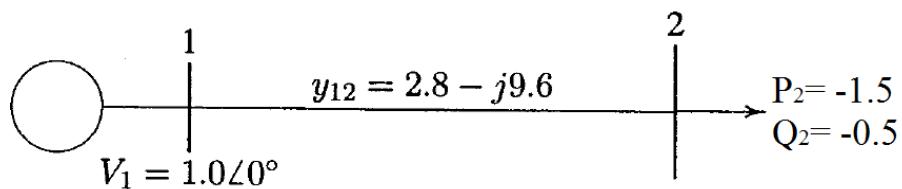
In the following two-bus system, the expression for real and reactive power at bus 2 is

$$P_2 = 10|V_2| \cos(\delta_2 - 106.26^\circ) + 10|V_2|^2 \cos(73.74^\circ)$$

$$Q_2 = 10|V_2| \sin(\delta_2 - 106.26^\circ) + 10|V_2|^2 \sin(73.74^\circ)$$

Using Newton Raphson method, determine the voltage magnitude and angle of bus 2.

Start with an initial estimate of  $|V_2|^{(0)} = 1$  and  $\delta_2^{(0)} = 0^\circ$ . Perform two iterations only.



Good luck

Power Systems 2  
 First Exam  
 First Semester 2017/2018  
Model Answers

Question 1

a)  $Y_{bus} = \begin{bmatrix} -j15 & j5 & 0 & j10 \\ j5 & -j15 & j10 & 0 \\ 0 & j10 & -j15 & j5 \\ j10 & 0 & j5 & -j15 \end{bmatrix}$

b)  $V_k^{(i+1)} = \frac{1}{Y_{kk}} \left[ \frac{P_k - jQ_k}{V_k^{(i)*}} - \sum_{n=1}^{k-1} Y_{kn} V_n^{(i+1)} - \sum_{n=k+1}^N Y_{kn} V_n^{(i)} \right]$

$$\begin{aligned}
 V_2^{(1)} &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{21} V_1 - Y_{23} V_3^{(0)} - Y_{24} V_4^{(0)} \right] \\
 &= \frac{1}{-j15} \left[ \frac{-0.5 + j0.2}{110^\circ} - j5 \times 1 - j10 \times 1 - 0 \right] \\
 &= 0.9872 \angle -1.93^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_3^{(1)} &= \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31} V_1 - Y_{32} V_2^{(1)} - Y_{34} V_4^{(0)} \right] \\
 &= \frac{1}{-j15} \left[ \frac{-0.4 + j0.1}{110^\circ} - 0 - j10 \times 0.9872 \angle -1.93^\circ - j5 \times 1 \right] \\
 &= 0.9857 \angle -2.84^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_4^{(1)} &= \frac{1}{Y_{44}} \left[ \frac{P_4 - jQ_4}{V_4^{(0)*}} - Y_{41} V_1 - Y_{42} V_2^{(1)} - Y_{43} V_3^{(1)} \right] \\
 &= \frac{1}{-j15} \left[ \frac{-0.2 + j0}{110^\circ} - j10 \times 1 - 0 - j5 \times 0.9857 \angle -2.84^\circ \right] \\
 &= 0.9953 \angle -1.7^\circ \text{ V}
 \end{aligned}$$

## Question 2

$$\frac{\partial P_2}{\partial \delta_2} = -10 |V_2| \sin(\delta_2 - 106.26^\circ)$$

$$\frac{\partial P_2}{\partial |V_2|} = 10 \cos(\delta_2 - 106.26^\circ) + 20 |V_2| \cos(73.74^\circ)$$

$$\frac{\partial Q_2}{\partial \delta_2} = 10 |V_2| \cos(\delta_2 - 106.26^\circ)$$

$$\frac{\partial Q_2}{\partial |V_2|} = 10 \sin(\delta_2 - 106.26^\circ) + 20 |V_2| \sin(73.74^\circ)$$

1st Iteration

$$\begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad J^{(0)} = \begin{bmatrix} 9.6 & 2.8 \\ -2.8 & 9.6 \end{bmatrix}$$

$$J^{(0)-1} = \begin{bmatrix} .096 & -.028 \\ -.028 & .096 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}^{(1)} = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}^{(0)} + J^{(0)-1} \left( \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix} - \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}_{\text{actual}} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} .096 & -.028 \\ -.028 & .096 \end{bmatrix} \begin{bmatrix} -1.5 \\ -.5 \end{bmatrix}$$

2nd iteration

$$= \begin{bmatrix} -.13 \\ .91 \end{bmatrix}$$

$$\begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}^{(1)} = \begin{bmatrix} -1.34 \\ -.38 \end{bmatrix}, \quad J^{(1)} = \begin{bmatrix} 8.33 & 1.07 \\ -3.66 & 8.32 \end{bmatrix}$$

$$J^{(1)-1} = \begin{bmatrix} .1136 & -.0147 \\ .05 & .1138 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}^{(2)} = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}^{(1)} + \begin{bmatrix} .1136 & -.0147 \\ .05 & .1138 \end{bmatrix} \left( \begin{bmatrix} -1.5 \\ -.5 \end{bmatrix} - \begin{bmatrix} -1.34 \\ -.38 \end{bmatrix} \right) = \begin{bmatrix} -.146 \\ .889 \end{bmatrix}$$